



## 25. On the Condition That the Roots of a Cubic Are Real or Imaginary

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∴ present value of an annuity of  $A$  for  $n$  years  
 = present value of a perpetual annuity of  $A$  - present value  
 of a perpetual annuity of  $A$  to commence in  $n$  years  

$$= \frac{A}{r} - \text{present value of } \frac{A}{r} \text{ due in } n \text{ years}$$

$$= \frac{A}{r} - \frac{A}{r} \times \frac{1}{R^n} = \frac{A(1 - R^{-n})}{r}.$$

PROF. LONEY.

25. On the condition that the roots of a cubic are real or imaginary.

Let one real root of the equation

$$x^3 + qx + r = 0$$

be  $2\alpha$ ; then the other roots may be written in the form  $-\alpha + \beta$ ,  $-\alpha - \beta$ , where  $\beta$  may be real or a pure imaginary.

Now  $q = -4\alpha^2 + \alpha^2 - \beta^2 = -3\alpha^2 - \beta^2$ ,  
 and  $r = 2\alpha(\alpha^2 - \beta^2)$ ,

$$\therefore 4q^3 + 27r^2 = -4\beta^2(9\alpha^2 - \beta^2)^2.$$

Now  $(9\alpha^2 - \beta^2)^2$  is positive whether  $\beta$  is real or imaginary, and  $\beta^2$  is positive or negative as the roots are real or imaginary.

Hence if  $4q^3 + 27r^2$  is positive, the roots are imaginary; if zero, equal; if negative, real.

T. G. VIVIAN.

26. On Continued Fractions.

Consider the continued fraction

$$a_1 + \frac{1}{a_2 + \frac{1}{a_3 + \dots + \frac{1}{a_n}}}$$

It is clear that, if  $p_n/q_n$  be the  $n^{\text{th}}$  convergent, both  $p_n$  and  $q_n$  are of the first degree in  $a_n$ ; it is also clear that no reduction to lower terms can take place in the course of the work.

Hence  $\frac{p_n}{q_n} = \frac{Aa_n + B}{Ca_n + D}$  .....(i.)

Now if  $a_n$  be made infinite, the convergent reduces to  $p_{n-1}/q_{n-1}$ .

Hence  $\frac{p_{n-1}}{q_{n-1}} = \frac{A}{C}$ ,

and  $A = p_{n-1}$ ,  $C = q_{n-1}$ . .....(ii.)

Again, if  $a_n$  be made zero, the convergent reduces to  $p_{n-2}/q_{n-2}$ .

Hence  $\frac{p_{n-2}}{q_{n-2}} = \frac{B}{D}$ ,

and  $B = p_{n-2}$ ,  $D = q_{n-2}$ ; .....(iii.)

therefore

$$p_n = a_n p_{n-1} + p_{n-2},$$

$$q_n = a_n q_{n-1} + q_{n-2}.$$

PROF. STEGGALL.

27. If  $\frac{p_n}{q_n}$  be the  $n^{\text{th}}$  convergent of the continued fraction  $\frac{a_1}{b_1 + \frac{a_2}{b_2 + \frac{a_3}{b_3 + \dots}}}$  it may always be assumed that  $p_0 = 0$ ,  $q_0 = 1$ .

In using the formulae  $p_n = b_n p_{n-1} + a_n p_{n-2}$ ,  $q_n = b_n q_{n-1} + a_n q_{n-2}$  for obtaining the successive convergents, it is always to be understood that if any convergent  $\frac{p_r}{q_r}$  is not in its lowest terms, it must not be reduced. The same condition applies to  $\frac{a_r}{b_r}$ . For the first two convergents we have

$$p_1 = a_1, \quad q_1 = b_1, \quad p_2 = a_1 b_2, \quad q_2 = b_1 b_2 + a_2.$$